



## A MODEL OF ACOUSTICAL RIGHT-ANGLE BEND (WITH METAMATERIALS)

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We propose a two-dimensional fan-shaped model to design an acoustical right-angle bend for sound beam. The quarter circular model is comprised of metamaterials with anisotropic refractive index. Due to the gradual decrease of the refractive index from inner to external along the radius of the model, the sound velocity increase with the increase of radius, which leads to a capability of bending a sound propagating beam through a right angle with no reflection loss. The results are validated by simulations in the frequency range of 3500 Hz to 4500 Hz. This model may have potential for practical applications in various areas such as sound absorption in pipeline and detection in industry.

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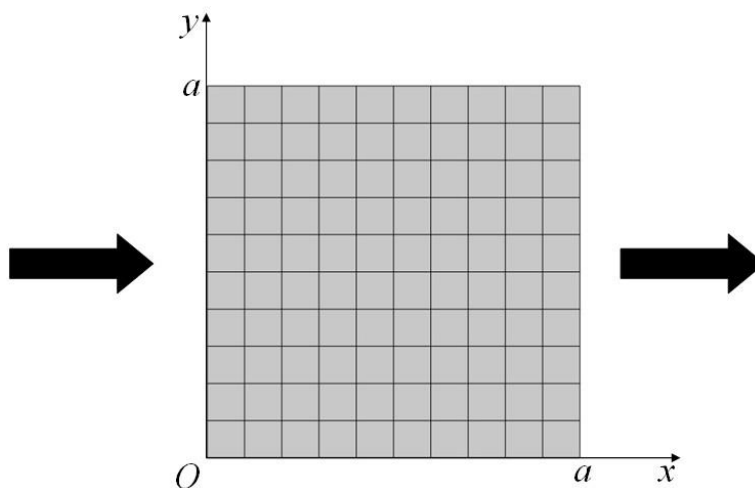
### 1. Introduction

Transformation acoustics (also called coordinate transformation) is a mathematical tool to obtain a transformed space from the original one. It provides a systematic method to control the propagation of sound waves. The coordinate transformation firstly comes from the electromagnetic field: J. B. Pendry<sup>[1]</sup> found that Maxwell's equations are form-invariant under different coordinate system. Then it quickly expanded to the sound field<sup>[2]</sup>. Till 2010, after a rapid development, Huanyang Chen and C. T. Chan<sup>[3]</sup> illustrated the transformation acoustics (TA) and then presented the parameters of invisibility cloak with the help of it. It offers a way to design the extraordinary metamaterial devices. Recently, many unprecedented devices have been obtained by combining the transformation acoustics with metamaterials. The most remarkable application is undoubtedly the acoustic cloaking<sup>[4]</sup>. Other applications like omnidirectional absorber<sup>[5]</sup>, acoustic GRIN lens<sup>[6]</sup>, extraordinary transmission<sup>[7]</sup> and subwavelength imaging<sup>[8]</sup> have also been proposed. The first three-dimensional broadband omnidirectional acoustic ground cloak<sup>[9]</sup> have been experimentally demonstrated lately, which demonstrate a practical use of transformation acoustics.

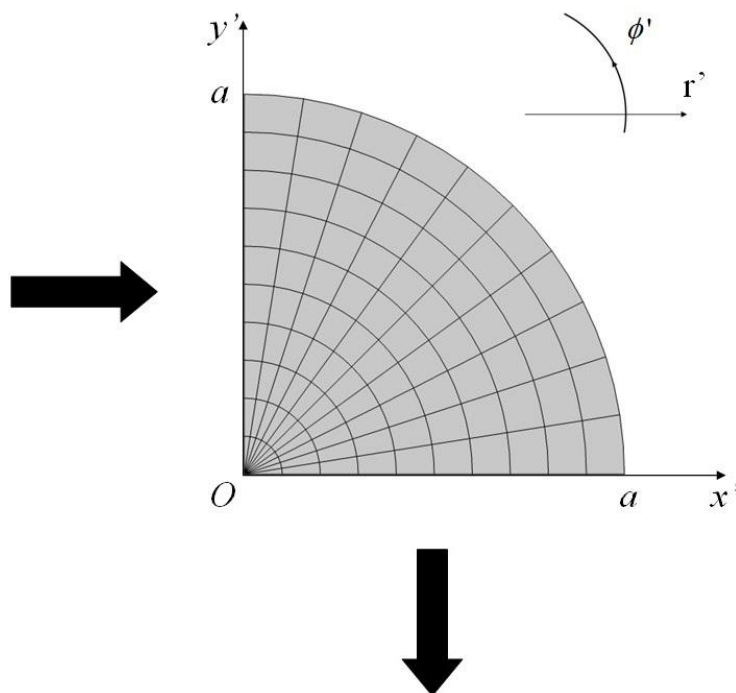
In this letter, we focus on a kind of beam shifter designed by transformation acoustics, namely, a two-dimensional model of acoustical right-angle bend. The first beam shifter and beam divider were presented by Marco Rahm, S. A. Cummer in 2008<sup>[10]</sup>. No matter the incoming beam incidented either normally or obliquely on the interface, the entire beam would be translated in another direction or divided into two beams. Do-Hoon Kwon and Douglas H Werner<sup>[11]</sup> then proposed a new way to bend the propagating beam. Based on the transformation optics, they proposed a model to rotate the direction of optical beam propagation by 90°. Finally we present a numerical design of an acoustical right-angle bend achieved by metamaterials with strong effective mass anisotropy.

## 2. Design

We simplify this problem to a two dimensional situation. A square region of size  $a \times a$  is shown in figure 1(a). Sound wave propagates from one side to another without reflection. This will be transformed into a fan-shaped region as shown in figure 1(b). After mapping figure 1(a) into figures 1(b), the sound beam from  $-x^2$ -direction will be bent to  $-y^2$ -direction without loss after passing through this model. That is to say, the propagating direction will be rotated by  $90^\circ$ .



**Figure 1(a).** The original coordinate system.



**Figure 1(b).** The transformed system.

The appropriate coordinate transformation we chosen is shown in figure 1, which can be written as:

$$r' = y. \quad (1)$$

$$\phi' = \frac{\pi}{2a}(a-x). \quad (2)$$

$$z' = z. \quad (3)$$

Where  $r' = \sqrt{x'^2 + y'^2}$ , and  $\phi' = \tan^{-1}\left(\frac{y'}{x'}\right)$ . Under this transformation, constant- $x$  lines in the Cartesian coordinates in figure 1(a) are correspond to constant- $\phi'$  lines in the polar coordinates in figure 1(b), and constant- $y$  lines are correspond to constant- $r'$  curves. Besides, the relationships between the Cartesian coordinates and the polar coordinates are  $x' = r' \cos \phi'$  and  $y' = r' \sin \phi'$ . Combining all of these formulas, we can obtain the Jacobian matrix of the coordinate transformation  $A = \left[ \frac{\partial(x', y', z')}{\partial(x, y, z)} \right]$ . According to the transformation acoustics<sup>[12,13]</sup>, the material parameter of the acoustical right-angle bend, i.e., mass density and bulk modulus, are  $\rho = \det(A)(A^{-1})^T A^{-1} \rho_0$ ,  $K = \det(A)K_0$ , where  $\rho_0 = 1.21 \text{kg} / \text{m}^3$  and  $K_0 = 0.14 \text{MPa}$  are the parameters of air.

$$\rho_{11} = \left( \frac{2a}{\pi r'} \sin^2 \varphi + \frac{\pi r'}{2a} \cos^2 \varphi \right) \rho_0. \quad (4)$$

$$\rho_{12} = \rho_{21} = \left( \frac{\pi r'}{4a} \sin 2\varphi - \frac{a}{\pi r'} \sin 2\varphi \right) \rho_0. \quad (5)$$

$$\rho_{22} = \left( \frac{\pi r'}{2a} \sin^2 \varphi + \frac{2a}{\pi r'} \cos^2 \varphi \right) \rho_0. \quad (6)$$

$$\rho_{33} = \frac{\pi r'}{2a} \rho_0. \quad (7)$$

$$K = \frac{\pi r'}{2a} K_0. \quad (8)$$

After a complicated calculating and simplification, we can eliminate  $\rho_{12}, \rho_{21}$  and obtain the key point of this structure:

$$\rho'_{11} = \frac{\pi r'}{2a} \rho_0. \quad (9)$$

$$\rho'_{22} = \frac{2a}{\pi r'} \rho_0. \quad (10)$$

$$\rho'_{33} = \frac{\pi r'}{2a} \rho_0. \quad (11)$$

$$K' = \frac{\pi r'}{2a} K_0. \quad (12)$$

Where  $\rho'_{11}, \rho'_{22}, \rho'_{33}$  and  $K'$  are the parameters after a rotation of the principal axis of the right-angle bend. The sound velocity can be solved by these parameters:

$$c'_{11} = c_0, c'_{22} = \frac{\pi r'}{2a} c_0, c'_{33} = c_0. \quad (13)$$

From the previous describe, we know that  $c'_{11}$  is mapped into constant- $\phi'$  lines and  $c'_{22}$  is mapped into constant- $r'$  lines, which means that sound velocity increase with the increase of radius. The outer velocity is much faster than the inner velocity in this model. This special property leads to a capability of bending a sound propagating beam through a right angle with no reflection loss.

### 3. Simulations

To test the effect of this right-angle bend, we bring a Gaussian beam to the sound field as the incident wave. The sound beam has its minimum width  $w_0$ . The solution of the time-harmonic Laplace's Equation for a 2D model, gives the following sound field ( $z$ -component):

$$P(x, y, z) = P_0 \sqrt{\frac{w_0}{w(x)}} e^{-\left(\frac{y}{w(x)}\right)^2} \cos\left(\omega t - kx + \eta(x) - \frac{ky^2}{2R(x)}\right) e_z. \quad (14)$$

Where

$$w(x) = w_0 \sqrt{1 + \left(\frac{x}{x_0}\right)^2}. \quad (15)$$

$$\eta(x) = \frac{1}{2} \arctan\left(\frac{x}{x_0}\right). \quad (16)$$

$$R(x) = x \left(1 + \left(\frac{x_0}{x}\right)^2\right). \quad (17)$$

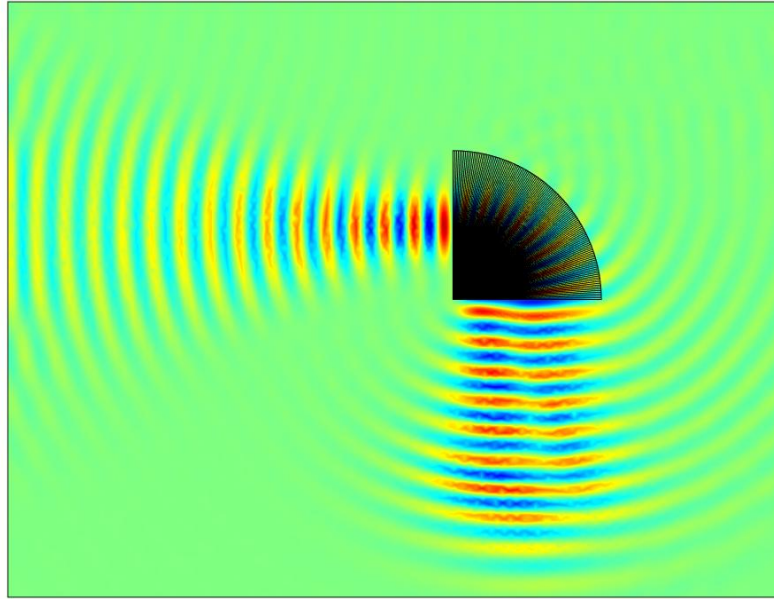
In these expressions,  $\omega$  is the angular frequency,  $y$  is the in-plane transverse coordination, and  $k$  is the wave number. The Gaussian beam is not a plane wave. It propagates like a spherical wave with radius  $R(x)$ . However, the wave front near the focal point is almost plane.

Now we need the acoustic layered system with two different kinds of isotropic materials (A and B) to realize the anisotropic properties of the model. The densities of these two isotropic materials are  $\rho_A$  and  $\rho_B$ , and the bulk modulus are  $K_A$  and  $K_B$ . The thickness of layer A and layer B are the same and they are much smaller than the wavelength of incident wave. Then we can obtain the effective density tensor and bulk modulus:

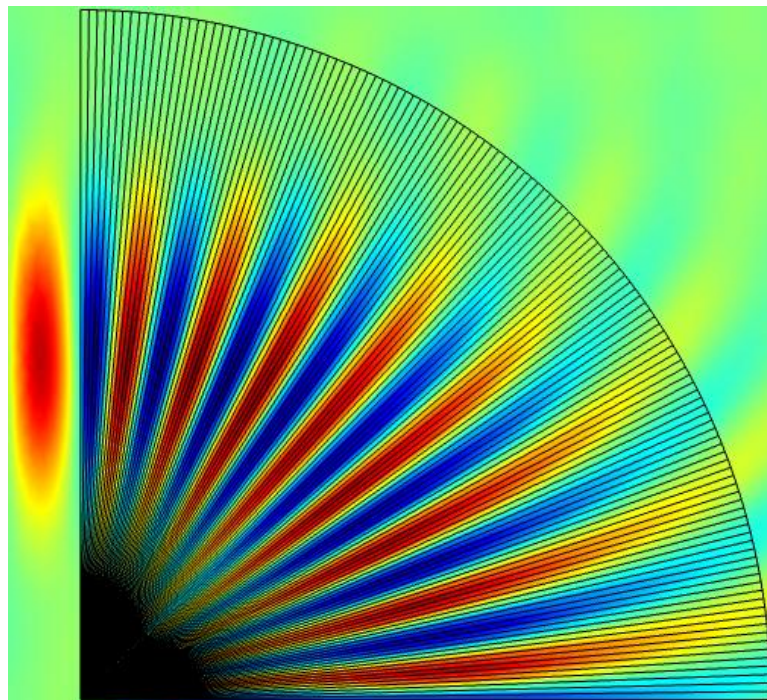
$$\rho_r = \frac{\rho_A + \rho_B}{2}, \rho_\phi = \frac{2\rho_A\rho_B}{\rho_A + \rho_B}, K = \frac{2K_A K_B}{K_A + K_B}. \quad (18)$$

The dimension of our model is  $a=0.5$  m, the minimum width of the sound beam is  $w_0=0.1$ m.

In figure 2(a), a 3500Hz Gaussian beam is incident on the right-angle bend. It is obviously that the direction of this propagating beam is rotated by  $90^\circ$  into the  $-y'$ -direction. We can see more details of the sound propagating in the model in figure 2(b). It is clearly that the the outer velocity is much faster than the inner velocity in this model and this property result in a bending of the sound beam.



**Figure 2(a).** A snapshot of the sound field.



**Figure 2(b).** Details of the sound propagating in the model.

#### 4. Discussion and conclusion

In summary, we have introduced a 2D structure to bend the Gaussian beam in the sound field. It seems impossible to realize these strange parameters in real life because  $\rho_{22} \rightarrow \infty$  when  $r' \rightarrow 0$ . However, actually  $r'$  don't need to be trend to 0. We can choose part of this model for realization. For example: the range of  $r'$  can be  $\frac{a}{\pi} \leq r' \leq \frac{2a}{\pi}$ , then  $\rho_0 \leq \rho_{22} \leq 2\rho_0$ . This will be much easier to achieve.

In fact, in reality, we focus more on the refractive index than the density and bulk modulus. That is to say, if the sound velocity satisfy this relationship:  $c'_{11} = c_0$ ,  $c'_{22} = \frac{\pi r'}{2a} c_0$ ,  $c'_{33} = c_0$ , the direction of propagating beam will be rotated by  $90^\circ$  in the model, no matter what the  $\rho$  and  $K$  will be. So we can realize this model by using layers of the perforated plates. However, we should notice that this method of using perforated plates will lead to impedance mismatching.

Actually this right-angle bend should be a broadband model. The upper limit of the effective frequency depends on the width between two layers. We only illustrate that the performance of this model is excellent from 3500Hz to 4500Hz, however, the range of the effective frequency could be wider.

In this paper, we propose an acoustical right-angle bend by utilizing the transformation acoustics. A 2D square was transformed into a fan-shaped region to achieve a capability of bending a sound propagating beam through a right angle without reflection. The realization may have potential application in various areas such as sound absorption in pipeline and detection in industry.

## REFERENCES

- <sup>1</sup> Pendry, John B., David Schurig, and David R. Smith. "Controlling electromagnetic fields." *science* 312.5781, 1780-1782, (2006).
- <sup>2</sup> Cummer, Steven A., et al. "Scattering theory derivation of a 3D acoustic cloaking shell." *Physical review letters* 100.2, 024301, (2008).
- <sup>3</sup> Chen, Huanyang, and C. T. Chan. "Acoustic cloaking and transformation acoustics." *Journal of Physics D: Applied Physics* 43.11, 113001, (2010).
- <sup>4</sup> Cummer, Steven A., and David Schurig. "One path to acoustic cloaking." *New Journal of Physics* 9.3, 45, (2007).
- <sup>5</sup> Li, Rui-Qi, et al. "A broadband acoustic omnidirectional absorber comprising positive-index materials." *Applied Physics Letters* 99.19, 193507, (2011).
- <sup>6</sup> Zigoneanu, Lucian, Bogdan-Ioan Popa, and Steven A. Cummer. "Design and measurements of a broadband two-dimensional acoustic lens." *Physical Review B* 84.2, 024305, (2011).
- <sup>7</sup> Lee, Sam Hyeon, et al. "Giant Acoustic Concentration by Extraordinary Transmission in Zero-Mass Metamaterials."
- <sup>8</sup> Cheng, Ying, et al. "Acoustic subwavelength imaging of subsurface objects with acoustic resonant metalens." *Applied Physics Letters* 103.22, 224104, (2013).
- <sup>9</sup> Zigoneanu, Lucian, Bogdan-Ioan Popa, and Steven A. Cummer. "Three-dimensional broadband omnidirectional acoustic ground cloak." *Nature materials* 13.4, 352-355, (2014).
- <sup>10</sup> Rahm, Marco, et al. "Optical design of reflectionless complex media by finite embedded coordinate transformations." *Physical Review Letters* 100.6, 063903, (2008).
- <sup>11</sup> Kwon, Do-Hoon, and Douglas H. Werner. "Transformation optical designs for wave collimators, flat lenses and right-angle bends." *New Journal of Physics* 10.11, 115023, (2008).
- <sup>12</sup> Norris, Andrew N. "Acoustic cloaking theory." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science* 464.2097, 2411-2434, (2008).
- <sup>13</sup> Popa, Bogdan-Ioan, Lucian Zigoneanu, and Steven A. Cummer. "Experimental acoustic ground cloak in air." *Physical review letters* 106.25, 253901, (2011).