

TKK Young Scientist Award in Mathematics and Physics

Solutions of Demailly's Strong Openness Conjecture and Related Problems in Several Complex Variables

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Prof. GUAN Qi'an
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The 2020 TKK Young Scientist Award in Mathematics & Physics went to Prof. GUAN Qi'an, a young talented mathematician from Peking University, for his solutions (joint with ZHOU Xiangyu) of a series of problems in several complex variables, particularly for his proof (joint with ZHOU Xiangyu) of the Demailly's strong openness conjecture.

GUAN has mainly engaged himself in the study of several complex variables, which explores the properties and structures of holomorphic functions of several variables, and is also called complex analysis of several variables. Because the properties of holomorphic functions are largely affected by the geometric and topological properties of their domains of definition, the research involves not only the studying of local properties, but also of global properties.

In the research of several complex variables, various

methods from partial differential equations, algebraic geometry, complex geometry, topology, Lie groups and other areas are widely applied. The research of function theory of several complex variables has also driven the development of the above-mentioned research fields. For example, LU Qikeng proved "LU Qikeng Theorem" named after him; SIU Yum-Tong proved the deformational invariance of plurigenera of projective algebraic manifolds; ZHOU Xiangyu proved the extended future tube conjecture which was listed as an unsolved problem in the *Encyclopedia of Mathematics*.

In cooperation with Prof. ZHOU Xiangyu, a CAS Member at the Academy of Mathematics and Systems Science (AMSS) under the Chinese Academy of Sciences, GUAN solved the optimal L^2 extension problem, proposing new ideas and methods. They established the optimal L^2 extension theorem by unifying the previous

various famous L^2 extension theorems; unveiling its unexpected connections with a lot of open problems in different fields, they further solved these problems.

Multiplier ideal sheaves play a central role in algebraic geometry. GUAN and ZHOU proved the strong openness conjecture on multiplier ideal sheaf associated to any plurisubharmonic function, which was posed by Prof. Demailly and thought to be “rather inaccessible”. This breakthrough was marked by a paper entitled “A proof of Demailly’s strong openness conjecture” appeared in the prestigious *Annals of Mathematics* in 2015.

“It solves an important problem in an unexpected and ingenious way,” commented the peer reviewers who handled the submitted manuscript for the journal of *Annals of Mathematics*. “In the reviewer’s opinion, the proofs of both the openness and the strong openness conjectures,” commented the *Mathematical Reviews*, “are among the greatest achievements in the intersection of complex analysis and algebraic geometry in recent years.”

The strong openness conjecture is a bottleneck

problem in the development of several complex variables and complex geometry. Many mathematicians have obtained important results under the assumption that the conjecture holds. Now, we know for sure that it holds.

GUAN and ZHOU also proved important conjectures posed by Demailly-Kollár and by Jonsson-Mustata, originated from complex geometric analysis and algebraic geometry.

Receiving his PhD under supervision of ZHOU in 2011 and working as a postdoctor in the Beijing International Center for Mathematics Research of Peking University for two years, he joined the School of Mathematical Sciences of Peking University in 2013, and is now a professor who enjoys teaching and illuminating young minds. As a young scholar trained in China, GUAN and his collaborators’ studies have brought new insights into the research field of several complex variables with a series of remarkable achievements.

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